*Int. J. Heat Mass Transfer.* Vol. 25, No. 7, pp. 917–923, 1982
Printed in Great Britain

0017-9310/82/070917-07 $03.00/0


# RADIATION HEAT TRANSFER USING MEAN PHOTON PATH LENGTHS

RICHARD O. BUCKIUS
Department of Mechanical and Industrial Engineering,
University of Illinois at Urbana-Champaign, Urbana, IL 61801, U.S.A.

(*Received* 23 *August* 1981 *and in revised form* 19 *October* 1981)

**Abstract**—An approximate approach to radiative heat transfer in an isotropically scattering and non-gray absorbing medium is presented. The absorption and scattering by the scattering constituents are gray and the gaseous non-gray absorption is banded. The approximate solutions are based upon mean photon path lengths of the photon path length distribution which is determined directly from the transport in a gray medium. The predictions for the radiative heat flux and intensity are shown to be accurate over large ranges in optical depth and scattering albedo.

## NOMENCLATURE

| | |
|---|---|
| $A_{i*}$, | wide band absorption; |
| $E_1(x)$, | first exponential integral; |
| $i^{\pm}$, | intensity; |
| $i_{i*}^{\pm}$, | scattering band absorption intensity; |
| $I^{\pm}$, | photon path length distribution for intensity; |
| $l$, | length of photon travel; |
| $L$, | layer thickness; |
| $q^{\pm}$, | heat flux; |
| $q_{i*}^{\pm}$, | scattering band absorption heat flux; |
| $Q^{\pm}$, | photon path length distribution for the heat flux; |
| $T$, | temperature; |
| $y$, | distance. |

Greek symbols

| | |
|---|---|
| $\alpha$, | absorption coefficient; |
| $\gamma$, | $= \alpha_\nu/(\alpha_S + \sigma)$; |
| $\gamma_E$, | Euler's constant; |
| $\eta_b$, | pressure broadening parameter; |
| $\theta$, | angle; |
| $\kappa$, | optical depth, $(\gamma + 1)\kappa_S$; |
| $\kappa_S$, | $= (\alpha_S + \sigma)y$; |
| $\kappa_H$, | optical depth at band head or center; |
| $\lambda$, | $(\alpha_S + \sigma)l$, photon path length; |
| $\mu$, | $= \cos\theta$; |
| $\langle\lambda\rangle^{\pm}_{\kappa_S}$, | mean photon path length at $\kappa_S$; |
| $\nu$, | wavenumber; |
| $\rho_{i*}$, | wide band absorption for reflection; |
| $\sigma$, | scattering coefficient; |
| $\tau_{i*}$, | wide band absorption for transmission; |
| $\omega_S$, | $= \sigma/(\alpha_S + \sigma)$; |
| $\omega_{bi}$, | exponential decay width. |

Subscripts

| | |
|---|---|
| b, | blackbody or band parameter; |
| H, | band head or center; |
| i, | band denotation; |
| L, | total layer thickness; |
| S, | scattering; |
| $\nu$, | wavenumber dependent; |
| *, | dimensionless. |

Superscript

| | |
|---|---|
| $\pm$, | positive or negative direction. |

## INTRODUCTION

COMPUTATIONS of radiation heat transfer in scattering media with highly non-gray absorption are generally complex and expensive. Radiant transfer in waters [1] shows very important spectral effects. The absorption and extinction of water is highly non-gray requiring very detailed spectral calculations. Transport in fog [2] requires similar detail due to the spectral dependence of water vapor and droplets. Fifty-two spectral intervals were used to compute the complete spectrum. The scattering contribution to the radiative transport including exponential wide band absorption has been investigated by numerical quadrature [3]. The internal and exiting intensities and radiative heat fluxes have been studied for a single wide band in the high pressure limit. The complexities increase significantly as the spectral detail increases and are particularly involved for banded gaseous absorption [3]. An alternate approach is to evaluate all possible paths lengths for the photons in a scattering medium and then determine the transport with non-gray absorption with the distribution [4, 5]. The advantage is that the path length distribution only needs to be evaluated once for the scattering medium and then any absorption can be included by a simple integration over path length. The major disadvantage of the path length approach is in the increased complexity of the solution for the path length distribution.

Approximate solutions vary from non-gray non-scattering predictions to gray scattering results [6–8]. An alternate approximate approach is presented in this work which is based upon the mean photon path length. Radiative transfer in a planar medium with exponential wide band absorption and isotropic scattering is investigated. A single wide band is studied although the results are directly applicable to multiband problems [3]. The mean path length concept was first discussed by Irvine [4] in regard to atmospheric line calculations and is developed here for heat transfer applications. The accuracy of the approximate solutions is assessed by comparing to exact solutions.

## PHOTON PATH LENGTH APPROACH

The system considered is a homogeneous plane layer with a diffuse source at the position zero and transparent conditions at $L$. The layer is composed of both gaseous and scattering components. The spectral gaseous absorption is banded and the gaseous scattering is neglected. The absorption and scattering by the scattering constituents is assumed to be gray and the scattering is isotropic. The layer optical depth based upon gaseous absorption at band head or center is denoted by $\kappa_{HL}$ and based upon the gray scattering constituents is $\kappa_{SL}$. (Note that $\kappa_{SL}$ includes gray absorption by the scattering constituents and $\kappa_{SL}$ of [5] includes only the scattering coefficient.) The albedo for the scattering components is denoted by $\omega_S$ and the relative significance of non-gray gaseous absorption and the gray extinction is expressed through $\gamma = \alpha_\nu/(\alpha_S + \sigma)$. The quantity $\gamma$ is required to incorporate spectral effects and is replaced by wide band absorption parameters after integration over a band.

The radiative heat flux and the intensity at a position $\kappa_S$ for a unit source at the boundary in a scattering medium (no spectral absorption) are denoted by $q_\nu^\pm(\kappa_S, \kappa_{SL}, \omega_S, \gamma = 0)$ and $i_\nu^\pm(\kappa_S, \mu, \kappa_{SL}, \omega_S, \gamma = 0)$, respectively. The contribution to the radiative heat flux from photons which have an optical path length for the scattering medium between $\lambda$ and $\lambda + d\lambda$ is denoted as $Q^\pm(\lambda, \kappa_S, \kappa_{SL}, \omega_S)d\lambda$. The length of photon travel $\lambda$ includes both the gray absorption and scattering contributions of the scattering constituents. The radiative heat flux resulting from all possible photon path lengths in a medium with only gray scattering and absorption is

$$q_\nu^\pm(\kappa_S, \kappa_{SL}, \omega_S, 0) = \int_0^\infty Q^\pm(\lambda, \kappa_S, \kappa_{SL}, \omega_S)d\lambda. \tag{1a}$$

Similar definitions for the intensity yield

$$i_\nu^\pm(\kappa_S, \mu, \kappa_{SL}, \omega_S, 0) = \int_0^\infty I^\pm(\lambda, \kappa_S, \mu, \kappa_{SL}, \omega_S)d\lambda. \tag{1b}$$

If the length of photon travel is denoted by $l$ then the gaseous absorption is included through $\exp(-\alpha_\nu l) = \exp(-\gamma\lambda)$. The resulting expression for the heat flux is

$$q_\nu^\pm(\kappa_S, \kappa_{SL}, \omega_S, \gamma) = \int_0^\infty e^{-\gamma\lambda}Q^\pm(\lambda, \kappa_S, \kappa_{SL}, \omega_S)d\lambda, \tag{2a}$$

and for the intensity is

$$i_\nu^\pm(\kappa_S, \mu, \kappa_{SL}, \omega_S, \gamma) = \int_0^\infty e^{-\gamma\lambda}I^\pm(\lambda, \mu, \kappa_{SL}, \omega_S)d\lambda. \tag{2b}$$

The diffuse source is taken to be black so that the total heat flux is

$$q^\pm(\kappa_S, \kappa_{SL}, \omega_S, \gamma) = \pi\int_0^\infty i_{\nu b}(T)q_\nu^\pm(\kappa_S, \kappa_{SL}, \omega_S, \gamma)d\nu, \tag{3a}$$

and the total intensity is

$$i^\pm(\kappa_S, \mu, \kappa_{SL}, \omega_S, \gamma) = \int_0^\infty i_{\nu b}(T)i_\nu^\pm(\kappa_S, \mu, \kappa_{SL}, \omega_S, \gamma)d\nu. \tag{3b}$$

The scattering band absorption properties are obtained by substituting equation (2) into equation (3) and expressing the gaseous absorption in terms of the wide band absorption. The exponential wide band absorption using a three parameter per band description is expressed as $A_{i*}(\kappa_H, \eta_b)$ where $\eta_b$ is the effective broadening parameter and the third parameter is the exponential decay width $\omega_{bi}$. Substituting equation (2a) into equation (3a) results in

$$q^\pm(\kappa_S, \kappa_{SL}, \omega_S, \gamma) = \pi\int_0^\infty Q^\pm(\lambda, \kappa_S, \kappa_{SL}, \omega_S)\left[\int_0^\infty i_{\nu b}(T)e^{-\gamma\lambda}d\nu\right]d\lambda, \tag{4}$$

and the term in the brackets is eliminated with

$$\int_0^\infty i_{\nu b}(T)e^{-\gamma\lambda}d\nu = i_b(T) - i_{\nu_i b}(T)\omega_{bi}A_{i*}(\kappa_H, \eta_b) \tag{5}$$

to yield

$$q_i^\pm(\kappa_S, \kappa_{SL}, \omega_S, \gamma) = \pi i_b(T)\int_0^\infty Q^\pm(\lambda, \kappa_S, \kappa_{SL}, \omega_S)d\lambda - \pi i_{\nu_i b}(T)\omega_{bi}\int_0^\infty Q^\pm(\lambda, \kappa_S, \kappa_{SL}, \omega_S) \times A_{i*}(\kappa_{HL}\lambda/\kappa_{SL}, \eta_b)d\lambda. \tag{6}$$

The subscript i denotes the single wide band considered. The first integral in equation (6) is obtained from equations (1a) and (3a) for the gray contribution. The scattering wide band absorption quantities, explicitly denoting the wide band parameters, are defined as

$$q_{i*}^\pm(\kappa_S, \kappa_{SL}, \omega_S, \kappa_{HL}, \eta_b) \equiv \frac{q^\pm(\kappa_S, \kappa_{SL}, \omega_S, 0) - q_i^\pm(\kappa_S, \kappa_{SL}, \omega_S, \gamma)}{\pi i_{\nu_i b}(T)\omega_{bi}} = \int_0^\infty Q^\pm(\lambda, \kappa_S, \kappa_{SL}, \omega_S)A_{i*}(\kappa_{HL}\lambda/\kappa_{SL}, \eta_b)d\lambda. \tag{7a}$$

Similar substitutions for the intensity yield

$$i_{i*}^\pm(\kappa_S, \mu, \kappa_{SL}, \omega_S, \kappa_{HL}, \eta_b) \equiv \frac{i^\pm(\kappa_S, \mu, \kappa_{SL}, \omega_S, 0) - i_i^\pm(\kappa_S, \mu, \kappa_{SL}, \omega_S, \gamma)}{i_{\nu_i b}(T)\omega_{bi}}$$

$$= \int_0^{\infty} I^{\pm}(\lambda, \kappa_S, \mu, \kappa_{SL}, \omega_S) A_{i*}(\kappa_{HL}\lambda/\kappa_{SL}, \eta_b)\, d\lambda. \quad (7b)$$

Note that these scattering band absorption quantities express the difference between the radiative transfer for the gray scattering constituents and gaseous wide band absorption. The radiative heat transfer including banded absorption is evaluated with the above quantities and the gray heat flux [3, 7]. The usefulness of these quantities for transport calculations has been presented in [3].

## MEAN PHOTON PATH LENGTH APPROACH

The heat flux path length distribution function, $Q^{\pm}(\lambda, \kappa_S, \kappa_{SL}, \omega_S)$ can be viewed as the probability that a photon which traverses an optical path length between $\lambda$ and $\lambda + d\lambda$ contributes to the radiative heat flux and is of a magnitude proportional to the heat flux in the gray scattering medium. The determination of the scattering wide band absorption properties in equations (7) requires the evaluation of the path length distribution. The mean value of this heat flux distribution function or the mean photon path length can be determined without evaluating the entire distribution. This mean is then used in the evaluation of the radiative heat transfer including the banded absorption.

In the linear region, the wide band absorption is

$$A_{i*}(\kappa_{HL}\lambda/\kappa_{SL}, \eta_b) = \kappa_{HL}\lambda/\kappa_{SL} \quad (8)$$

so that equation (7a) yields

$$q_{i*}^{\pm}(\kappa_S, \kappa_{SL}, \omega_S, \kappa_{HL}, \eta_b) = \frac{\kappa_{HL}}{\kappa_{SL}} \int_0^{\infty} \lambda Q^{\pm}(\lambda, \kappa_S, \kappa_{SL}, \omega_S)\, d\lambda. \quad (9)$$

The mean photon path length of the heat flux path length distribution function or the mean path length is defined as

$$\langle \lambda \rangle_{\kappa_S}^{\pm} = \int_0^{\infty} \lambda Q^{\pm}(\lambda, \kappa_S, \kappa_{SL}, \omega_S)\, d\lambda / q_v^{\pm}(\kappa_S, \kappa_{SL}, \omega_S, 0)$$

$$= \frac{-1}{q_v^{\pm}(\kappa_S, \kappa_{SL}, \omega_S, 0)} \frac{\partial}{\partial \gamma} [q_v^{\pm}(\kappa_S, \kappa_{SL}, \omega_S, \gamma)]_{\gamma \to 0} \quad (10)$$

or, alternatively, as

$$\langle \lambda \rangle_{\kappa_S}^{\pm} = -\frac{\partial}{\partial \gamma} \{\ln [q_v^{\pm}(\kappa_S, \kappa_{SL}, \omega_S, \gamma)]\}_{\gamma \to 0}. \quad (11)$$

With this mean path length definition, the scattering band absorption heat flux is given as

$$q_{i*}^{\pm}(\kappa_S, \kappa_{SL}, \omega_S, \kappa_{HL}, \eta_b) = q_v^{\pm}(\kappa_S, \kappa_{SL}, \omega_S, 0)\kappa_{HL}\langle \lambda \rangle_{\kappa_S}^{\pm}/\kappa_{SL}$$

$$= q_v^{\pm}(\kappa_S, \kappa_{SL}, \omega_S, 0) A_{i*}(\kappa_{HL}\langle \lambda \rangle_{\kappa_S}^{\pm}/\kappa_{SL}, \eta_b). \quad (12a)$$

This will be an exact calculation in the linear region of the wide band absorption and only an approximation in the other limits. The corresponding expression for the intensity using the intensity mean is

$$i_{i*}^{\pm}(\kappa_S, \mu, \kappa_{SL}, \omega_S, \kappa_{HL}, \eta_b) = i_v^{\pm}(\kappa_S, \mu, \kappa_{SL}, \omega_S, 0) \times A_{i*}(\kappa_{HL}\langle \lambda \rangle_{\kappa_S}^{\pm}/\kappa_{SL}, \eta_b). \quad (12b)$$

These expressions reveal that the radiative transport with gray absorption and scattering for the scattering components and non-gray gaseous absorption is evaluated directly from the radiative transport with gray absorption and scattering. The mean path length is evaluated directly from the radiative heat flux without the need for the complete distribution. The mean is the substituted in equation (12) to evaluate any wide band absorption. The mean in equations (10) and (11) is for the heat flux path length distribution yet a similar expression is applicable for the mean of the intensity path length distribution. No special notation is used to differentiate between the means for intensity and heat flux.

## RESULTS AND DISCUSSION

The solution for the radiative transfer for all gray components has been obtained by the method of successive approximations. The singular integrands were evaluated by the method of singularity subtraction [9] and the integrations were performed with a quadrature of order 40 [10]. The derivative in equation (9) was evaluated with a 3-point, equally spaced difference expression. Therefore, the determination of the mean photon path length requires three evaluations of the radiative transfer for the gray medium. The approximate solutions using the mean path length approach will be compared to the solutions presented in [3] for the high pressure limit and those presented in [5] for other values of pressure broadening.

The mean photon path lengths have been evaluated for the positions $\kappa_S = \kappa_{SL}$ (transmission) and $\kappa_S = 0$ (reflection) for four different scattering albedos. Figure 1 presents the results in terms of the quantity $\langle \lambda \rangle / \kappa_{SL}$ which represents the mean photon path length compared to the total scattering optical depth or as multiples of the total scattering optical depth. An alternate interpretation of this quantity is the mean geometric photon path length compared to the physical length of the medium ($\langle l \rangle / L$).

The mean path lengths for transmission, $\langle \lambda \rangle_{\kappa_{SL}}^{+}$, are always greater than $\kappa_{SL}$ since the minimum path length is $\kappa_{SL}$. The conservative scattering curve shows an increase of the mean quantity with $\kappa_{SL}$ resulting from the broader photon path length distribution that exists for larger $\kappa_{SL}$. The mean quantity for transmission at small albedos shows a decrease with $\kappa_{SL}$ due to the increase in gray absorption and, therefore, the depletion of the longer path lengths. Note that the mean geometric photon path length is increasing in all the above cases but the presented mean quantity is a

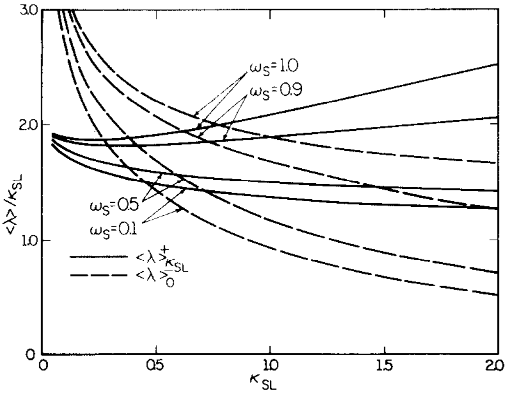


FIG. 1. Heat flux mean photon path lengths for transmission and reflection.

relative length to the layer thickness which decreases with gray absorption.

The mean quantities for reflection, $\langle\lambda\rangle_0^-$, reflect the fact that the photon path length distribution has components with zero path length. The mean exhibits a large number of multiples of $\kappa_{SL}$ when the layer depth is relatively thin and a decrease in the number of multiples of $\kappa_{SL}$ as the medium becomes optically thick. The effect of the absorption through the scattering albedo is similar to the results for transmission.

The mean path lengths are substituted directly into equation (12a) to determine wide band absorption quantities with scattering. The wide band absorption applicable in the high pressure limit is [8]:

$$A_{i*}(\kappa_H, \eta_b) = A_{i*}(\kappa_H) = \ln(\kappa_H) + E_1(\kappa_H) + \gamma_E \quad (13)$$

and the Felske–Tien correlation [11] is used at other pressure broadening values. These expressions are the same as those employed in the exact calculations which are used for comparison. The reflection $[\rho_{i*}(\kappa_{SL}, \omega_S, \kappa_{HL}) = q_{i*}^-(0, \kappa_{SL}, \omega_S, \kappa_{HL})]$ and transmission $[\tau_{i*}(\kappa_{SL}, \omega_S, \kappa_{HL}) = q_{i*}^+(\kappa_{SL}, \kappa_{SL}, \omega_S, \kappa_{HL})]$ results using the mean path lengths are presented in Figs. 2–4.

The results presented are interpreted with equation (12a) by noting that the scattering band absorption quantities are the multiple of the heat flux in a scattering medium and the wide band absorption based upon the mean path length. The small albedo results for transmission (Fig. 2) indicate large decreases as $\kappa_{SL}$ increases. This is due to a large decrease in the transmission heat flux as $\kappa_{SL}$ increases coupled with a decrease in the relative mean path length (and therefore the wide band absorption). The conservative scattering result for transmission does not show as large a decrease with increasing $\kappa_{SL}$ since the relative mean path length increases to counter the decrease in the gray scattering heat flux. The reflection band absorption (Fig. 3) is strongly dependent upon the scattering contribution since the quantity would be zero if $\omega_S = 0$. The relative mean photon path length for reflection decreases with increasing $\kappa_{SL}$ for all albedos which tends to decrease the absorption. The radiative heat flux always increases with increasing $\kappa_{SL}$. These competing effects indicate an increase in the

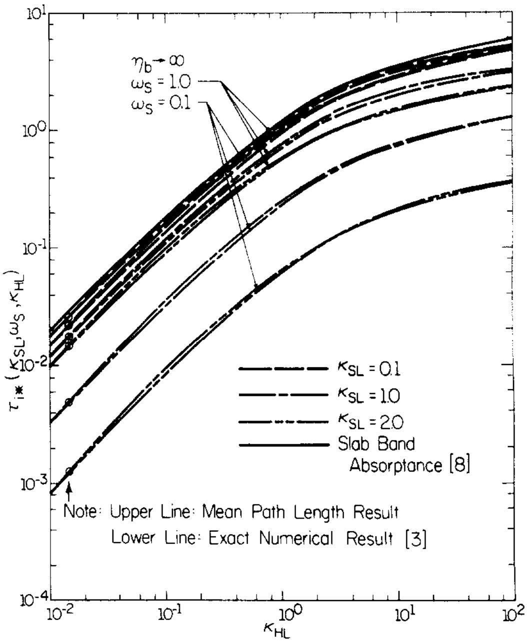


FIG. 2. Hemispherical wide band absorption for transmission ($\eta_b \to \infty$).

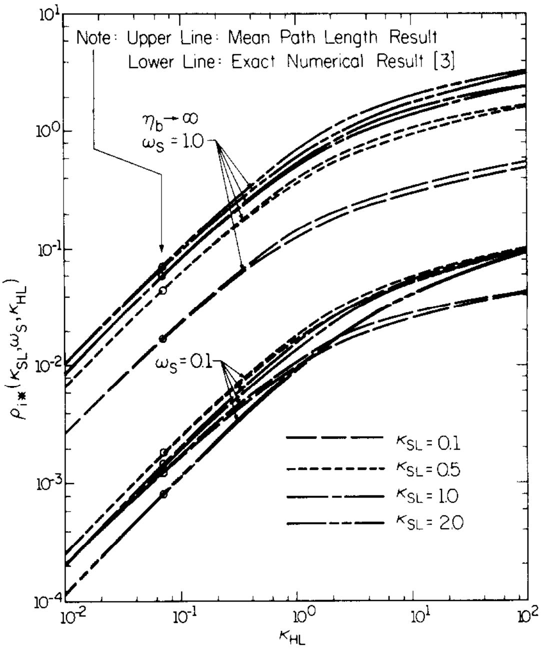


FIG. 3. Hemispherical wide band absorption for reflection ($\eta_b \to \infty$).

wide band absorption for reflection with conservative scattering and a crossing over for smaller albedos. This figure also indicates a shifting of the transition from linear to logarithmic asymptotes to larger $\kappa_{HL}$ as $\kappa_{SL}$ increases.

The scattering band absorption quantities determined by both the approximate and exact approach are the same for the small values of $\kappa_{HL}$. This is expected from the development in equation (12). The approximate solution predicts both the trends and magnitudes accurately over all values of $\kappa_{SL}$, $\kappa_{HL}$ and $\omega_S$. The approximate solution is also accurate for small pressure broadening parameters as indicated in Fig. 4. The error in the approximate approach is greater for the transmission results which indicates that a single mean parameter is unable to exactly include all effects.

The mean photon path lengths for the intensity are presented in Figs. 5 and 6. The top portion of each figure presents the transmission means and the bottom presents the reflection means. The transmission results for conservative scattering (Fig. 5) show large mean photon path lengths for large layer optical depths in the normal direction. As the transmission angle increases for a specific $\kappa_{SL}$, the mean path lengths increase. These increases are a result of the increased optical depth for the layer in the normal direction and for the line of sight at other angles for a specified $\kappa_{SL}$. The small layer optical depth results indicate a large increase in the relative mean photon path lengths with large angles which are almost entirely composed of direct radiation (non-scattered) and, therefore, quite long at grazing angles. The mean photon path lengths for reflection increase with angle for small optical depths and decrease with angle for large optical depths. The large relative means for small $\kappa_{SL}$ results from the importance of 1-order scattering while the decrease in the relative mean for large $\kappa_{SL}$ is an optically thick type dependence. Note again that the results are presented as the ratio of the mean to the layer thickness so that the actual mean length does increase as $\kappa_{SL}$ increases.

The typical effects of the scattering albedo on the intensity means for transmission and reflection are presented in Fig. 6. The means generally decrease as the albedo decreases as a result of the depletion of the long path lengths of the distribution. The smaller optical depths for the case of transmission indicate a different trend at large angles where the small albedo prediction shows a large relative mean resulting from the dominance of directly transmitted energy.

The intensity predictions from the mean path length approach are compared to the exact results by the high pressure limit in Fig. 7. The angular effects for transmission $[\tau'_{i*}(\mu, \kappa_{SL}, \omega_S, \kappa_{HL}) = i^+_{i*}(\kappa_{SL}, \mu, \kappa_{SL}, \omega_S, \kappa_{HL})]$ and reflection $[\rho'_{i*}(\mu, \kappa_{SL}, \omega_S, \kappa_{HL}) = i^-_{i*}(0, \mu, \kappa_{SL}, \omega_S, \kappa_{HL})]$ are predicted very accurately by the mean path length approach. The small optical depth results show large changes in absorption with angle since the relative mean for intensity varied significantly with angle (Fig. 5). Note that these results are normalized with respect to the normal value of the respective quantity.

The mean path length approach also predicts the distributions throughout the medium accurately. The mean path lengths are presented in Fig. 8 and the wide band absorption heat flux results are given in Fig. 9. The mean path lengths for the positive heat flux monotonically increase with $\kappa_S$. The slopes are larger for the conservative scattering case than for the small albedo prediction as noted earlier. The mean path lengths for the negative flux are relatively constant throughout the medium. The means for both directions are greater than a unit slope since the path length distribution is zero between the path lengths of zero and $\kappa_S$. The wide band absorption heat flux is given as $q_{i*}(\kappa_S, \kappa_{SL}, \omega_S, \kappa_{HL}) = q^+_{i*}(\kappa_S, \kappa_{SL}, \omega_S, \kappa_{HL}) - q^-_{i*}(\kappa_S, \kappa_{SL}, \omega_S, \kappa_{HL})$ and the results are compared in Fig. 9. The results indicate the accuracy of the approximate approach as well as the varied dependences resulting from scattering. The slab band absorptance [8] is the non-scattering result.

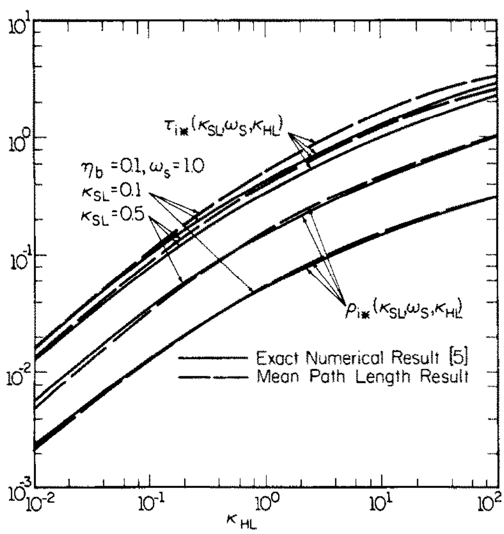


FIG. 4. Hemispherical wide band absorption for transmission and reflection ($\eta_b = 0.1$).

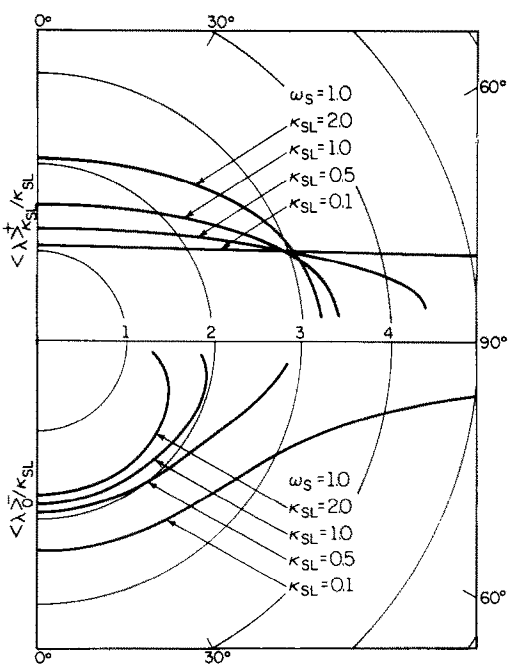


FIG. 5. Intensity mean photon path lengths for transmission and reflection.

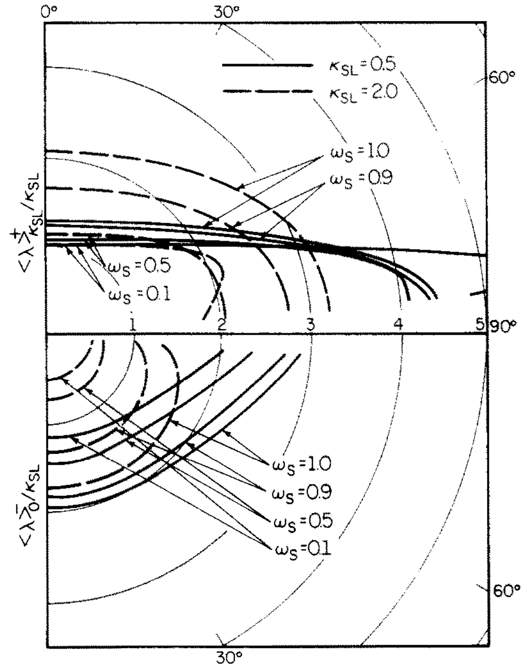


FIG. 6. Intensity mean photon path lengths for transmission and reflection.

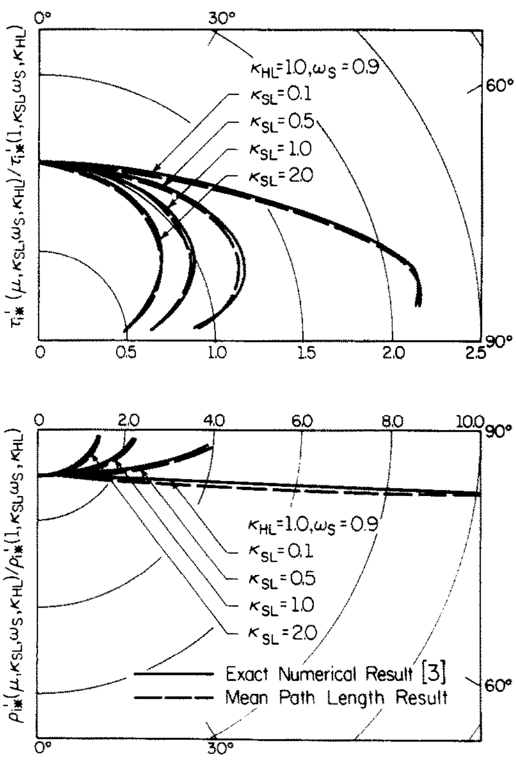


FIG. 7. Intensity wide band absorption for transmission and reflection.

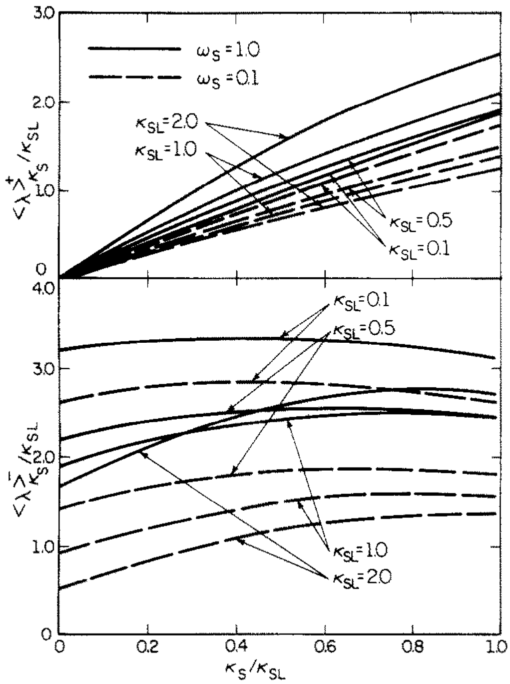


FIG. 8. Heat flux mean photon path lengths throughout the medium.

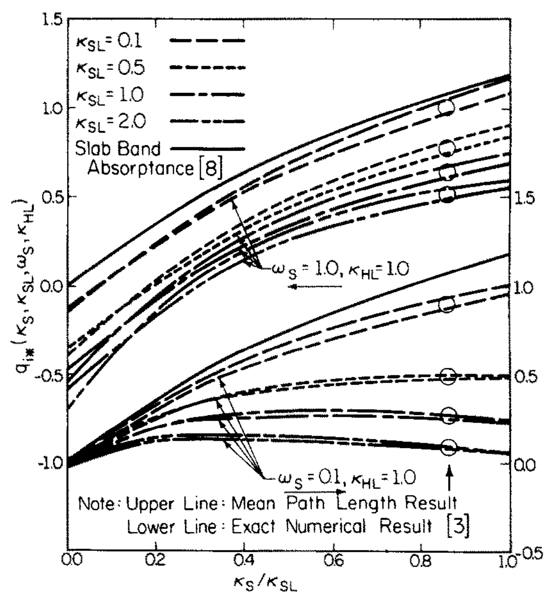


FIG. 9. Wide band absorption heat flux throughout the medium.

## CONCLUSIONS

An approximate approach has been presented to quantify radiation heat transfer in a scattering and non-gray absorbing medium. The approach is based upon mean photon path lengths of the path length distributions. The mean path length is determined directly from the radiation transport in a gray medium without evaluating the path length distribution. The mean path length requires three evaluations of the radiative transport for a gray medium. The mean path length is then used in equation (12) to evaluate the radiative heat transfer. A quadrature calculation for a single wide band may require up to 25 evaluations [3]. In multi-band calculations, the computational advantage of the mean path length is significant since only the mean must be determined (three evaluations of the radiative transport) as compared to a set of quadrature points for each band. The mean path length approach produces exact answers in the linear limit and approximate yet accurate results in the other regions.

*Acknowledgement*—This work was supported in part by the National Science Foundation (MEA 81-09250).

## REFERENCES

1. R. Viskanta and J. S. Toor, Radiant energy transfer in waters, *Water Resources Res.* **8**, 595–608 (1972).
2. G. Korb and W. Zdunkowski, Distribution of radiative energy in ground fog, *Tellus* **12**, 298–320 (1970).
3. R. O. Buckius, The effect of molecular gas absorption on radiative heat transfer with scattering, submitted for publication (1981).
4. W. M. Irvine, The formation of absorption bands and the distribution of photon optical paths in a scattering atmosphere, *Bull. Astron. Inst. Neth.* **17**, 266–279 (1964).
5. R. O. Buckius and A. Fernandez-Fraga, The optical path length approach to radiation heat transfer with isotropic scattering and gaseous absorption, *J. quant. Spectros. radiat. Transfer* **24**, 1–13 (1980).
6. D. K. Edwards, Molecular gas band radiation, *Adv. Heat Transfer* **12**, 115–193 (1976).
7. A. L. Crosbie, Apparent radiative properties of an isotropically scattering medium on a diffuse substrate, *J. Heat Transfer* **101**, 68–75 (1979).
8. D. K. Edwards and A. Balakrishnan, Slab band absorptance for molecular gas radiation, *J. quant. Spectros. radiat. Transfer* **12**, 1379–1387 (1972).
9. D. R. Hartree, *Numerical Analysis*. Oxford University Press, London (1958).
10. A. S. Kronrod, *Nodes and Weights of Quadrature Formulas*. Consultants Bureau, New York (1965).
11. J. D. Felske and C.-L. Tien, A theoretical closed form expression for the total band absorbance of infrared radiating gases, *Int. J. Heat Mass Transfer* **17**, 155–158 (1974).

## TRANSFERT THERMIQUE PAR RAYONNEMENT ET PARCOURS MOYEN DES PHOTONS

**Résumé**—On présente une approche du transfert thermique par rayonnement dans un milieu absorbant, non gris et diffusant isotropiquement. L'absorption et la diffusion par les constituants diffusants sont grises et l'absorption par le gaz non gris se fait par bandes. Les solutions approchées sont basées sur le parcours moyen des photons qui est déterminé directement à partir du transfert dans un milieu gris. Le calcul du flux thermique radiatif et de l'intensité est précis sur un large domaine de profondeur optique et d'albedo.

## WÄRMEAUSTAUSCH DURCH STRAHLUNG BEI VERWENDUNG DER MITTLEREN PHOTONENWEGLÄNGE

**Zusammenfassung**—Es wird eine Näherungsmethode des Strahlungsaustausches in einem isotrop streuenden und nicht-grau absorbierendem Medium vorgestellt. Die Absorption und die Streuung der streuenden Anteile sind grau, und die im Gas stattfindende nicht-graue Absorption erfolgt in Banden. Die Näherungslösungen basieren auf der mittleren Weglänge der Photonen, die sich aus der Weglängenverteilung ergibt, welche unmittelbar aus dem Transport in einem grauen Medium bestimmbar ist. Die Bestimmung des strahlungsbedingten Wärmestroms und seiner Intensität sind über weite Bereiche der optischen Tiefe und des streuenden Albedos genau.

## ОПИСАНИЕ ЛУЧИСТОГО ТЕПЛООБМЕНА НА ОСНОВЕ СРЕДНЕЙ ДЛИНЫ ПРОБЕТА ФОТОНА

**Аннотация** — Представлен приближенный метод расчета лучистого теплопереноса в несерой поглощающей среде с изотропным рассеянием. Поглощение и рассеяние на соответствующих компонентах являются серыми, а для несерого поглощения газа определена полоса. Приближенные решения основаны на средней длине пробега фотонов, определяемой по распределению длин пробега для серой среды. Показано, что результаты расчетов плотности и интенсивности лучистого теплового потока являются достаточно точными в широком диапазоне оптической глубины и альбедо рассеяния.